Planning of UWB Indoor Positioning Network Using Binary Integer Linear Programming

***Abstract*—The Ultra Wide Band (UWB) is a promising indoor positioning sensor. Its precise range measurement and wall penetration capability are well suited for indoor navigation. UWB anchor locations must also be carefully determined to achieve the optimal performance of a UWB positioning. However, previous approaches on the UWB anchor placement resulted in poor positioning accuracy around corners of an indoor environment and ignored UWB wall penetration effects. To overcome those shortcomings, this paper introduces an advanced method of an UWB anchor network design that provides high positioning accuracy in all user space and minimizes uncertainty due to UWB signal wall penetrations with a binary integer linear programming. It is expected that the presented method would provide a reliable positioning performance that is needed for the navigation of autonomous systems.**

***Index Terms*—UWB, GPS, indoor positioning, sensor location.**

I. Introduction

THE Ultra Wide Band (UWB) is one of the popular systems for indoor navigation and tracking. A UWB positioning system consists of a user tag and several anchors fixed at known coordinates. The anchors are typically time-synchronized and the user transmits a sharp pulse to the anchors. Then one of the anchors or a master station gathers the time stamps of the received signals and formulates time difference of arrival (TDOA) measurements to compute the user position [1] [2]. For navigation, a master station sends back the computed position to the user. Another architecture is that a user tag measures the range between the user and an anchor through two way communications. In this architecture, a user tag computes its own position using time of arrival (TOA) formulation [3]. Due to the extremely narrow pulse width within a few nanoseconds, UWB is able to provide precise ranging accuracy better than a few centimeters. Also, its wall penetration capability is another attractive feature for reliable indoor positioning.

To achieve a better positioning performance using UWB, it is important to analyze and understand the range error characteristics of UWB in an indoor environment. Previous research in this regard includes the UWB range accuracy modeling with respect to the signal travel distance and bandwidth of a UWB pulse and wall penetration excess delay [4] [5]. In addition, positioning algorithms are another factor that effects UWB positioning performance [6] [7]. However, another important or even more crucial part of an optimal positioning is an anchor network geometry.

Previously, an anchor network geometry has been optimized from using regular geometries [8] [9], meta-heuristics or heuristics [10], genetic algorithm [11] [12], or Cramer-Rao bounds [13] [14]. The approaches based on regular or fixed geometries typically use a square or triangle lattice. The meta-heuristic approach in [10] uses an objective function that balances precision and non-availability. In this method, an optimal positioning network is found through Diversified Local Search that attempts to find a solution with the minimum number of anchors. Reference [11] uses a genetic algorithm-based approach to obtain an optimal configuration of ultrasonic sensors for 3 dimensional positioning. The cost function in this approach is the absolute value of the determinant of the design matrix in TDOA formulation. The genetic algorithm in reference [12] rather uses Dilution of Precision (DOP) values of the user positioning area in its fitness function to search for an optimal network in a complex environment. Reference [13] uses the Cramer-Rao bound to find an optimal ranging source configuration that minimizes the variance of the mean squared error. In this method, the ranging source configuration is required to be symmetrically spread with respect to a user. More recent approaches are to optimize the internode distance of anchors [15] [16] using the criterion of minimizing the average mean square error (MSE) in the estimated user position. More useful algorithms can also be found in other fields such as structural health monitoring since an optimal sensor network is imperative to efficiently detect structure failures [17]–[20].

An indoor positioning network planned from the previous approaches exhibits two important drawbacks. First, the users near the corners of a building would experience relatively lower positioning accuracy. The reason is that the fixed or symmetric anchor geometry is able to only provide good positioning accuracy to the users near the center of the anchor geometry. In general, the maximum positioning accuracy is obtained at the center of the network where a user is able to see a large number of anchors with various azimuth angles. The users around the corners may likely be outside the anchor geometry and would see the relatively lower number of anchors with a limited azimuth angle variation, thus suffering from a poor positioning accuracy. When an autonomous system navigates inside a building, the positioning accuracy around corners must be guaranteed to avoid collision. Another drawback is that the prior arts did not take into account additional range errors occurred when the UWB signal penetrates walls. When there are a number of rooms in a building, the number of wall penetrations from an anchor to a user quickly increases and the range accuracy correspondingly deteriorates. If this additional range error is neglected in planning an indoor positioning network, a user would experience lower positioning accuracy than expected and may not be able to achieve given missions.

This paper proposes heuristics-based binary integer linear programming (BILP) algorithms to overcome the two drawbacks in planning an indoor UWB positioning network. In this approach, the BILP is formulated with some proposed heuristics that enforce the BILP to efficiently find a (local) optimal indoor anchor geometry. The resultant geometry minimizes the number of wall penetrations and provides required positioning accuracy to the users around the corners of a building. The paper first overviews UWB range measurement and positioning error models. Then, we discuss the BILP formulation tailored for indoor positioning and demonstrate its optimization performance using a given floor plan.

II. UWB Range Measurement and Positioning Error Model

A. UWB Range Error Model

The UWB range measurements between a user tag and an anchor are subject to some errors. The main range error sources can be categorized into multipath, thermal noise, and wall penetration excessive delay [21]. Then, the UWB range measurement between a user and the i^{th} anchor can be modeled as follows

$$r_i = d_i + b_i + \epsilon_i \tag{1}$$

where r_i is the range measurement. d_i is the true range to the i^{th} anchor. b_i is a bias and is mainly caused by the excessive delay through walls. ϵ_i is a zero mean Gaussian random noise that includes multipath and thermal noise.

The standard deviation of the Gaussian random noise, σ_ϵ, depends on the bandwidth of the UWB pulse, B, and the distance that the pulse travels, d. With this characteristics, σ_ϵ is modeled as follows [4]

$$\sigma_\epsilon(B,d) = \sigma_0 g_B(B) g_d(d) + \sigma_0 \tag{2}$$

with:

$$g_B(B) = g_1 e^{(-B/g_2)} \tag{3}$$

$$g_d(d) = d^\alpha \tag{4}$$

where the coefficients of g_1, g_2, α, and σ_0 are listed in Table I.

The bias magnitude depends on the relative electrical permittivity, γ, of the objects that a UWB signal penetrates through. It has been also shown in [4] that the magnitude of the bias linearly increases with the number of the objects. Assuming that the walls are made of the same material, then the bias is modeled as [4] [5]

$$b = n_w \times b_o \tag{5}$$

$$= n_w \times (\sqrt{\gamma} - 1)\frac{d_w}{c} \tag{6}$$

where n_w is the number of the walls that the UWB signal penetrates through. b_o is the excessive delay of a wall. d_w is the thickness of the wall and c is the speed of light.

The modeled bias and random noise will be used to generate range errors in the simulated indoor environment later in this paper.

TABLE I
Parameters for a UWB noise uncertainty model (LOS: Line of sight, NLOS: Non-Line of sight)

	$\sigma_0[m]$	α	g_1 $[m^{-\alpha}]$	g_2 [GHz]
LOS	0.016	1.5	0.64	0.60
NLOS	0.049	1.5	0.21	0.73

B. Positioning Uncertainty Analysis

Although both TOA and TDOA based UWB systems are possible, the analysis of positioning uncertainty in this section assumes a TDOA based positioning system. However, the analysis of the TOA based UWB system would be similar to a TDOA based system. Let us assume that there are m anchors in a UWB positioning network and they are well time-synchronized. The coordinate of the i^{th} anchor is denoted as $\boldsymbol{a}_i = (a_{i,x}, a_{i,y}, a_{i,z})$. Taking $\boldsymbol{a}_1$ as the reference node, then a TDOA equation at user location, $\boldsymbol{x}_u$, can be formulated as

$$f(\boldsymbol{a}, \boldsymbol{x}_u) = \begin{bmatrix} r_2 - r_1 \\ r_3 - r_1 \\ \vdots \\ r_m - r_1 \end{bmatrix} = \begin{bmatrix} \|\boldsymbol{x}_u - \boldsymbol{a}_2\| - \|\boldsymbol{x}_u - \boldsymbol{a}_1\| \\ \|\boldsymbol{x}_u - \boldsymbol{a}_3\| - \|\boldsymbol{x}_u - \boldsymbol{a}_1\| \\ \vdots \\ \|\boldsymbol{x}_u - \boldsymbol{a}_m\| - \|\boldsymbol{x}_u - \boldsymbol{a}_1\| \end{bmatrix} \tag{7}$$

Then, the positioning uncertainty due to random noise, σ_ϵ, can be modeled as [22]

$$\sigma_p = \sqrt{\text{trace}(\boldsymbol{H}^\top \boldsymbol{W} \boldsymbol{H})^{-1}} \tag{8}$$

where H is

$$\boldsymbol{H} = \begin{bmatrix} \dfrac{\boldsymbol{x}_u - \boldsymbol{a}_2}{\|\boldsymbol{x}_u - \boldsymbol{a}_2\|} - \dfrac{\boldsymbol{x}_u - \boldsymbol{a}_1}{\|\boldsymbol{x}_u - \boldsymbol{a}_1\|} \\ \dfrac{\boldsymbol{x}_u - \boldsymbol{a}_3}{\|\boldsymbol{x}_u - \boldsymbol{a}_3\|} - \dfrac{\boldsymbol{x}_u - \boldsymbol{a}_1}{\|\boldsymbol{x}_u - \boldsymbol{a}_1\|} \\ \vdots \\ \dfrac{\boldsymbol{x}_u - \boldsymbol{a}_m}{\|\boldsymbol{x}_u - \boldsymbol{a}_m\|} - \dfrac{\boldsymbol{x}_u - \boldsymbol{a}_1}{\|\boldsymbol{x}_u - \boldsymbol{a}_1\|} \end{bmatrix} \tag{9}$$

and W is

$$\boldsymbol{W} = \begin{bmatrix} \sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2 & \sigma_{\epsilon_1}^2 & \cdots & \sigma_{\epsilon_1}^2 \\ \sigma_{\epsilon_1}^2 & \sigma_{\epsilon_1}^2 + \sigma_{\epsilon_3}^2 & \cdots & \sigma_{\epsilon_1}^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{\epsilon_1}^2 & \sigma_{\epsilon_1}^2 & \cdots & \sigma_{\epsilon_1}^2 + \sigma_{\epsilon_m}^2 \end{bmatrix}^{-1} \tag{10}$$

When there exist some biases, Δb, in the range measurements, the induced positioning bias, $\boldsymbol{P}_B$, is

$$\boldsymbol{P}_B = (\boldsymbol{H}^\top \boldsymbol{W} \boldsymbol{H})^{-1} \boldsymbol{H}^\top \boldsymbol{W} \Delta \boldsymbol{b} \tag{11}$$

In Eq. (8), the position uncertainty depends on two factors: range measurement uncertainty and the user-anchor geometry. The quality of the user-anchor geometry is commonly quantized as Dilution of Precision (DOP) that is defined as

$$\text{DOP} = \sqrt{\text{trace}(H^\top H)^{-1}} \tag{12}$$

The favorable user-anchor geometry provides the lower value of DOP that reduces the position uncertainty in Eq. (8) and the position bias in Eq. (11).

III. UWB Indoor Positioning Network Optimization Algorithm

An optimal anchor network geometry for a UWB positioning system could be defined as the one that meets a user positioning accuracy requirement with the minimum number of anchors. The anchor geometry determination is basically a facility location problem and is classified as NP-complete in computational complexity theory. In a NP-complete problem, there are no known general efficient algorithms that output an exact solution(s) other than an (almost) exhaustive search. Therefore, when faced with the NP-complete problems, a typical approach is to find an approximation rather than seeking an exact solution by using heuristics that can be applied to a particular problem [23] [24]. Depending on the chosen heuristics, the solutions could be a local optimum.

A. Baseline Approaches of Anchor Placement

In a TDOA positioning system, a user requires at least three anchors for 2D horizontal positioning and four anchors for 3D positioning. In addition, to achieve a better positioning accuracy with a fixed number of ranging sources, the ranging sources in view at a user position should have some angular diversity. One way to achieve this with a lower number of anchors is to keep some distance between the anchors. For 2D positioning, the angular diversity in the horizontal plane is sufficient. However, 3D positioning requires angular diversity of the anchors in the vertical plane as well. These baseline approaches are mathematically formulated in a Binary Integer Linear Programming (BILP).

To prepare for a BILP formulation, the possible anchor and user locations are first divided into many small grids. Then, a unique index number is assigned to each grid. With the grids, the BILP consists of an objective function and constraints, which are all linear. The objective function is the weighted sum of the number of chosen anchors, and the constraints have various conditions and heuristics. Then, the BILP optimization process minimizes the value of the objective function while meeting the constraints and the required positioning accuracy. The lower value of the objective function tends to use the lesser number of anchors, which leads to more efficient anchor networks. The motivation of using the BILP is that it can immediately determine if a given total number of anchors is enough for a user to be able to compute its position in all user positioning area irrespective of a required positioning accuracy. This eliminates a significant infeasible set of the anchor configuration space in its search procedure. The heuristics further help to reduce the search space and to find an optimal solution more efficiently. Overall, there are two stages of the BILP formulation and they search for an optimal solution through iterations [25]. The details of the BILP optimization process are discussed in the next subsection.

B. Formulation of Binary Integer Linear Programming

The Binary Integer Linear Programming (BILP) for an optimal UWB anchor placement can be formulated as:

$$\begin{aligned} \text{minimize } Z &= \sum_{i=1}^{N} w_i p_i = \boldsymbol{w}^\top \boldsymbol{p} \\ \text{subject to: } \boldsymbol{V}\boldsymbol{p} &\geq \boldsymbol{q} \\ \boldsymbol{D}\boldsymbol{p} &\leq \boldsymbol{g} \\ \boldsymbol{A}\boldsymbol{p} &\leq \boldsymbol{h} \\ \boldsymbol{w}^\top \boldsymbol{p} &\leq Z_{min} \\ p_i &\in \{0,1\} \end{aligned} \tag{13}$$

where Z is the cost function to be minimized and $\boldsymbol{p}$ is the grid index vector of the candidate anchor locations. $\boldsymbol{p}$ takes the binary value of either 0 or 1. When $p_i = 1$, it contains an anchor at the i^{th} anchor location. Otherwise, $p_i = 0$. Vector $\boldsymbol{w}$ is a weighting factor on $\boldsymbol{p}$. If every candidate anchor location is equally treated, $\boldsymbol{w}$ has the same value in all vector elements such as 1s.

Matrix $\boldsymbol{V}$ in Eq. (13) is a visibility matrix. The i^{th} row of $\boldsymbol{V}$ corresponds to a grid index of a user location and the j^{th} column to an anchor location. The elements of matrix $\boldsymbol{V}$ also take on the value of either 0 or 1. If a user at the i^{th} row location sees the anchor at the j^{th} column location, $\boldsymbol{V}_{ij}$ is equal to 1. Otherwise, $\boldsymbol{V}_{ij}$ is equal to 0. The vector $\boldsymbol{q}$ is the required minimum number of visible anchors at the corresponding user location.

Matrix $\boldsymbol{D}$ and vector $\boldsymbol{g}$ control the minimum separation between anchors. If the distance between i and j anchors is less than the minimum separation, $\boldsymbol{D}_{ij}$ is equal to 1. Otherwise, $\boldsymbol{D}_{ij}$ is equal to 0. The vector $\boldsymbol{g}$ designates the allowed number of anchors inside the separation limit at each candidate anchor location. Matrix $\boldsymbol{A}$ contains the previous solution sets denoted as $\boldsymbol{p}_s$ at each row, and the vector $\boldsymbol{h} = (\boldsymbol{p}_s^\top \boldsymbol{1} - S_N)\boldsymbol{1}$ forces the BILP formulation to yield a unique solution during the iterative search. S_N is a non zero positive integer number. Finally, Z_{min} is the minimum cost among the valid solution sets found through previous iterations. Thus the last constraint, $\boldsymbol{w}^\top \boldsymbol{p} \leq Z_{min}$, reduces the search space such that the BILP only looks for a new solution having a cost less than or equal to Z_{min}. Therefore, the resultant user accuracy is evaluated once the solution from Eq. (13) is obtained. The BILP is solved by using GNU Linear Programming Kit [26].

In this proposed formulation, the optimization is performed on the space of discrete sensor grid locations rather than continuous locations as the prior arts in [8] [9] [10]. The reason for that is that the BILP and heuristics can be more conveniently applied in discrete locations than continuous locations.

When the first BILP does not satisfy the required position accuracy in all user position, the second BILP is implemented using the following formulation

$$\begin{aligned} \text{minimize } Z &= \sum_{i=1}^{N} w_i p_i = \boldsymbol{w}^\top \boldsymbol{p} \\ \text{subject to: } \boldsymbol{V}\boldsymbol{p} &\geq \boldsymbol{q}_\text{s} \\ \boldsymbol{D}_\text{s}\boldsymbol{p} &\leq \boldsymbol{g}_\text{s} \\ \boldsymbol{A}_\text{s}\boldsymbol{p} &\leq \boldsymbol{h}_\text{s} \\ \boldsymbol{w}^\top\boldsymbol{p} &\leq Z_{min} \\ p_i &\in \{0,1\} \end{aligned} \tag{14}$$

and

$$q_{s,i} = \begin{cases} q_{s,i}+1 & \text{if accuracy is not met} \\ q_{s,i} & \text{if accuracy is met} \end{cases} \tag{15}$$

This second step of Eq. (14) is repeated until a solution set meeting the required position accuracy is found or no solution is feasible due to the conflicts among the constraints. In the second BILP, the user location grid having insufficient positioning accuracy is required to see one additional ranging source at each iteration. This constraint is included in $\boldsymbol{q}_\text{s}$. Matrix D_s relaxes the separation between anchors at small steps for each iteration to allow additional anchors to be placed near the chosen anchor positioning in the previous iteration. Matrix A_s has the solution sets in the second step of the BILP formulation.

When the search space is small or the required positioning accuracy is loose, the second stage of BILP does not need to be implemented. In fact, the first stage can only be used in general and would result in the same performance. However, when the search space becomes large and the required positioning accuracy is tight, the search time by using the first stage BILP only would significantly increase because the linear programming solver has to solve a large set of linear equations at once. When the second stage BILP is incorporated, the complexity of the linear equation is divided into the first and second stage BILPs. Now, the first and second stage BILPs each need to solve a smaller set of linear equations. Also, the partial solution from the first stage BILP is treated as fixed or determined anchor locations in the second stage BILP. The computation time in this approach is overall faster than using the first stage BILP only.

The block diagram in Fig. 1 shows the overall network node search algorithm using the BILPs. To demonstrate the performance of the algorithm, it will be tested with a targeted indoor environment in the next section.

C. Additional Heuristics for Indoor Positioning

The BILP formulations in Eq. (13) and Eq. (14) could be augmented with the following heuristics to take into account the users at the edges of a building, additional ranging errors due to wall penetrations, and 3D positioning: weighting on sensor locations, averaged wall penetration number, and vertical anchor height variation.

1) Weighting on Outer Edge Sensor Locations: In general, a user near the outer edges or corners would have the most limited anchor network geometry. Anchors on the outer edges may be necessary to provide a decent anchor angular diversity

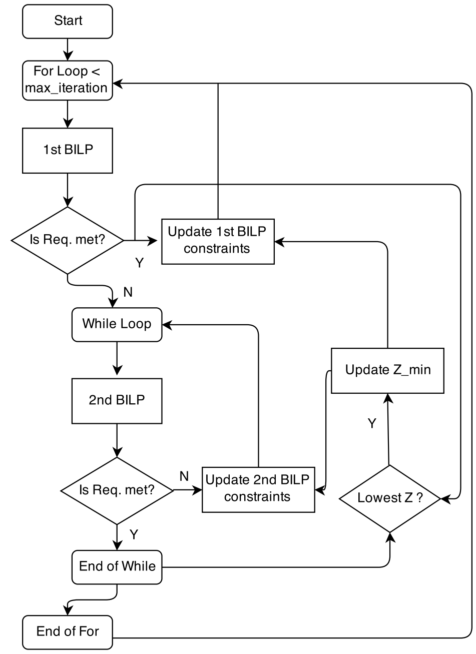


Fig. 1. Network node planning search algorithm using BILPs.

to those difficult user locations. The anchor placement at the outer edges can be enforced in the BILPs by using the weighting vector, $\boldsymbol{w}$, as follows

$$w_i = \begin{cases} 1 & \text{if } x_i \text{ is on outer edge} \\ 1+w_o & \text{if } x_i \text{ is NOT on outer edge} \end{cases} \tag{16}$$

where w_o is a positive number.

2) Averaged Wall Penetration Number: When the wall penetration delay cannot be compensated and its contribution to the positioning error is significant, it is desirable to place anchors in the form that induces lesser wall penetrations in the user space. For this purpose, the objective function can be altered as follows:

$$\text{minimize } Z = \boldsymbol{w}^\top\boldsymbol{p} + \frac{\boldsymbol{1}^\top\boldsymbol{T}\boldsymbol{p}}{M} \tag{17}$$

where $\boldsymbol{T}$ is a wall penetration matrix. The row of $\boldsymbol{T}$ corresponds to the user locations, and the column to the anchor locations. The matrix element of T_{ij} has the number of walls between the user at i^{th} location and the anchor at j^{th} location. M is the total number of the user space grids. This objective function in Eq. (17) not only attempts to minimize the number of anchors but also reduces the number of wall penetrations in the user space.

3) Vertical Anchor Height Variation: For a good 3D localization performance, the height of the anchors in the network must vary. A simple strategy for this can be formulated as follows:

$$\boldsymbol{c}^\top\boldsymbol{p} - \boldsymbol{f}^\top\boldsymbol{p} = |k| \tag{18}$$

and

$$c_i = \begin{cases} 0 & \text{if } p_i \text{ is NOT on the ceiling} \\ 1 & \text{if } p_i \text{ is on the ceiling} \end{cases} \tag{19}$$

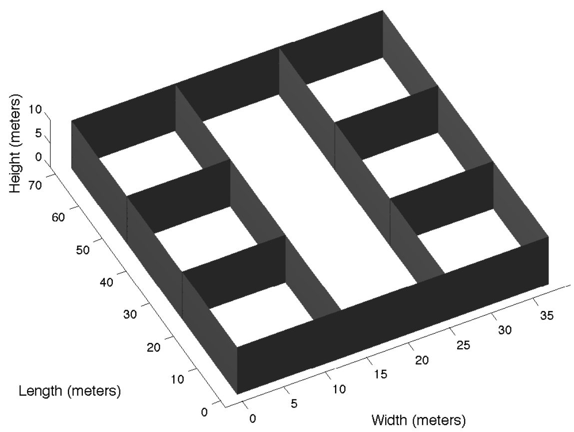
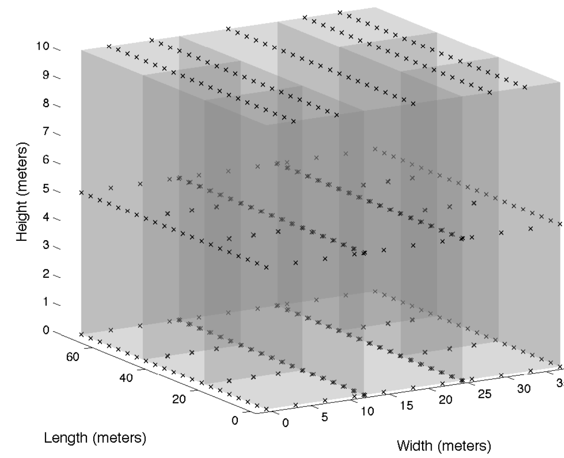
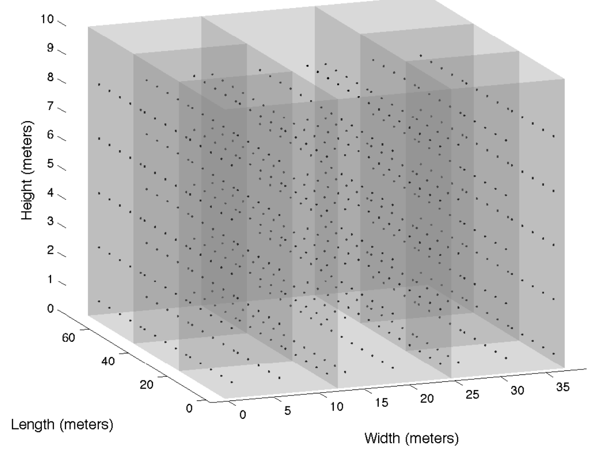


Fig. 2. Floor plan of a targeted indoor environment.



(a)



(b)

Fig. 3. (a) Small 'x's indicate candidate sensor locations in the building. (b) Dots indicate user location grids for the evaluation of position accuracy.

and

$$f_i = \begin{cases} 0 & \text{if } p_i \text{ is NOT on the floor} \\ 1 & \text{if } p_i \text{ is on the floor} \end{cases} \tag{20}$$

This constraint drives the difference of the number of anchors placed on the floor and the ceiling to be equal to k. When k is set to 0, then the same number of anchors is placed on the floor and the ceiling.

IV. Performance Assessment of the Optimal Anchor Planning Algorithm

A. Indoor Environment Set-Up

The performance of the proposed algorithm was assessed through simulation with the given indoor environment in Fig. 2. The indoor space has 70 m long and 37.5 m width. The height from the floor to the ceiling is set to 10 m. As shown in Fig. 2, there is one long hall way as well as six rooms which are separated by walls. The coordinate system in the indoor environment has the origin at the lower left corner.

The anchor placement area may include the floor, walls, and ceiling. Fig. 3 (a) shows the candidate anchor locations that are distributed in the following three levels: floor, ceiling, and mid-level between the ceiling and floor. An anchor on the walls may be placed either inside or outside of the room. In this example, no objects like furniture are considered. However, if there are objects that lead to severe line of sight obstructions, the beacons on the floor should be avoided.

Fig. 3 (b) shows the grid points of user navigation space generated from the floor plan in Fig. 2. Note that the user positioning accuracy will be evaluated in those grid points.

B. Anchor Placements for 2D and 3D positioning

The UWB devices are assumed to have the bandwidth of 7.5 GHz. The Gaussian random noise are simulated based on Eq. (2) and Table I. When there are walls between the user and anchor grids, the NLOS parameters in Table I are used. With 7.5 GHz bandwidth, the noise standard deviation is between 1.6 to 5 cm. The RF coverage of an anchor is assumed to be 50 m, which seems to be adequate from the authors' survey of today's commercial UWB devices.

The simulation ran for the following four cases: 2D horizontal with wall effects, 2D horizontal without wall effects, 3D with wall effects, and 3D without wall effects. The cases without wall effects are considered because the excessive delay due to wall penetration can be well modeled and compensated. In some other cases, the excessive delay may be negligible for the materials of the walls whose γ is close to 1. The simulation with negligible wall effects uses the objective function, Z, of Eq. (13), and the cases with wall effects uses Eq. (17). The simulation in the former case would search for the anchor network that meets the specified position accuracy requirement with the minimum number of anchors. However, the latter case would search for an anchor network that balances the number of anchors and the averaged number of wall penetrations in the user space to reduce the unknown position bias due to the wall penetrations.

In the simulation, a user was required to see more than 3 anchors for 2D and 4 anchors for 3D positioning. The

TABLE II
THE RESULTANT OPTIMAL UWB ANCHOR NETWORKS FOR 2D POSITIONING (NA: NUMBER OF ANCHORS, AWP: AVERAGE WALL PENETRATIONS)

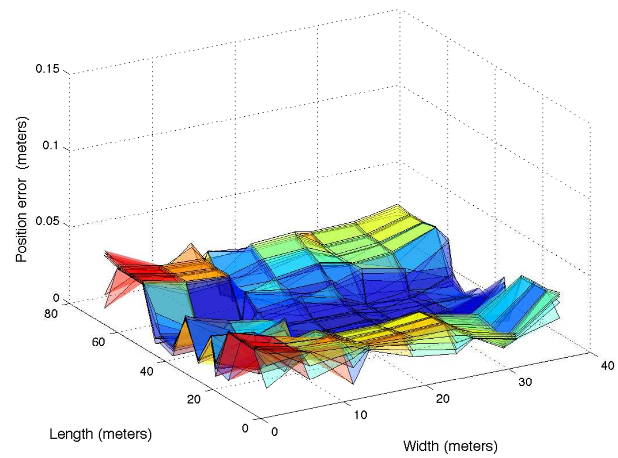
	2D w/o walls			2D w/ walls		
Thresholds	NA	AWP	rms(σ_p)	NA	AWP	rms(σ_p)
$\sigma_{\text{thr}} = 10$ cm	5	7.59	6 cm	5	7.59	6 cm
$\sigma_{\text{thr}} = 7$ cm	7	10.93	4 cm	8	10.8	4 cm
$\sigma_{\text{thr}} = 4$ cm	12	17.1	1 cm	12	16.72	3 cm

TABLE III
THE RESULTANT OPTIMAL UWB ANCHOR NETWORKS FOR 3D POSITIONING (NA: NUMBER OF ANCHORS, AWP: AVERAGE WALL PENETRATIONS)

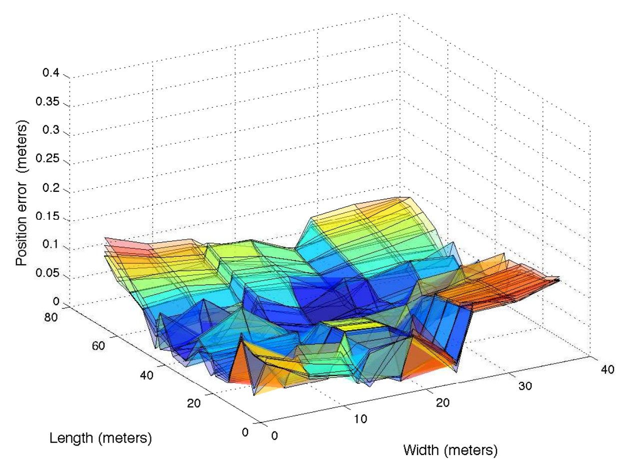
	3D w/o walls			3D w/ walls		
Thresholds	NA	AWP	rms(σ_p)	NA	AWP	rms(σ_p)
$\sigma_{\text{thr}} = 30$ cm	7	10.4	12 cm	8	9.73	11 cm
$\sigma_{\text{thr}} = 20$ cm	8	12.17	10 cm	8	12.11	10 cm
$\sigma_{\text{thr}} = 15$ cm	11	16.58	8 cm	11	15.98	6 cm

minimum distances among the anchors varied from 7 meters to 17 meters. The value of S_N for the vector $\boldsymbol{h}$ was set to 4. In 3D positioning, the difference of the number of anchors in the floor and ceiling, k, was set to 1. To give a more preference on the outer edges of the building, the parameter of w_o was set to 1. The positioning uncertainty of Eq. (8) is required less than a given accuracy threshold of σ_{thr} in the entire user space. The accuracy thresholds were set to 4 cm, 7 cm, and 10 cm for 2D positioning and Table II lists the simulation results in the number of anchors (NA), averaged wall penetrations (AWP), and rms of σ_p in the user space. The minimum accuracy requirement for 3D positioning was set to 15 cm, 20 cm, and 30 cm and the corresponding results are listed in Table III. The anchor locations for each case are shown in Appendix. Fig. 4 (a) and Fig. 4 (b) show the resultant positioning accuracy of the anchor networks for σ_{thr} = 4 cm in 2D without walls and σ_{thr} = 15 cm in 3D without walls. The user positioning accuracy was evaluated at the user height of 0.5, 2.38, 4.25, 6.13, and 8 meters and superimposed in the figures. These two examples were chosen to show that σ_{thr} is met in the entire user spaces of the anchor networks.

In the simulation results, the 2D positioning required 5 to 12 anchors and the 3D positioning required 7 to 11 anchors in the given indoor environment. As expected, the tighter σ_{thr} is, the more number of anchors were selected. The number of required anchors was overall similar regardless of wall effects. However, the cases taking into account wall effects induce smaller AWP and tend to place anchors in the middle hallway. This is a reasonable outcome because the anchors in the middle hallway would induce lesser wall penetrations than the anchors placed on the outer edges. A good example is the cases of σ_{thr} = 20 cm and σ_{thr} = 30 cm in 3D with walls of Table III. Interestingly, both cases choose the same number of anchors but have different anchor network configurations. When σ_{thr} = 30 cm, five anchors are placed in the middle hallway as shown in Fig. 8 in Appendix. As σ_{thr} is tightened to 20 cm, only one anchor remains in the hallway and four anchors move toward to the outer edges to further reduce DOPs in the user space while keeping the same number of anchors. This increases the overall wall penetrations, which is inevitable to meet the required σ_{thr}.



(a)



(b)

Fig. 4. Positioning accuracy of the optimal positioning network without wall effects (a) 2D σ_{thr} = 4 cm (b) 3D σ_{thr} = 0.15 cm.

C. Discussion of Optimality and Execution Time

As stated before in Section III, the use of heuristics may result in a local optimum. Nevertheless, the optimality of the BILP solution must be checked to see the goodness of the resultant anchor network. However, this is not an easy task because the only direct method for this is an exhaustive search that would take enormous time and is not feasible. However, the optimality can be inferred from the number of anchors seen in the user space in relatively simple cases. For example, as shown in Fig. 5 , when $\sigma_{\text{thr}} = 10$ cm, a user was able to see 3 anchors in most user locations, which is the minimum number of anchors required for 2D positioning. For the case of 3D with $\sigma_{\text{thr}} = 30$, a user sees only 4 anchors in the large part of the user space. Again, a user needs at least 4 anchors to compute its 3D position. Therefore, Fig. 5 implies that the resultant

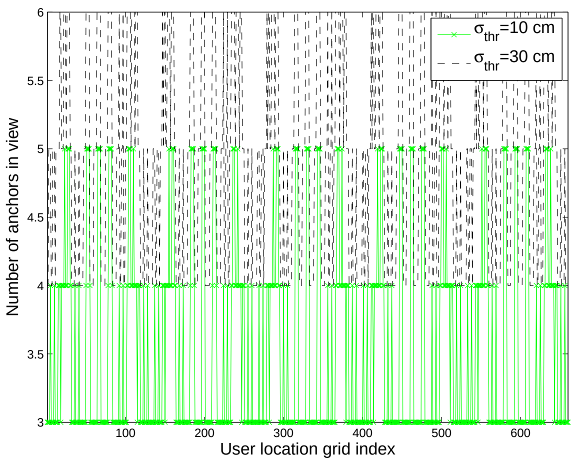


Fig. 5. The number of anchors in view at user location grids in 2D ($\sigma_{\text{thr}} = 10$ cm) and 3D ($\sigma_{\text{thr}} = 30$ cm) without walls.

network in the two cases should be close to the optimum. Based on this observation, the cases for more stringent σ_{thr} are also expected to be optimal and further analysis is left for future research.

With a modern personal computer having an 1.3 GHz processor, the algorithms found the optimal solutions within ten minutes except for the stringent cases of σ_{thr} = 4 cm in 2D and σ_{thr} = 15 cm in 3D. The incorporation of the wall penetration does not effect the overall execution time. In those stringent σ_{thr} cases, the execution time took about two hours.

V. Conclusion

This paper introduced a (local) optimal UWB anchor node placement algorithm. The algorithm uses Binary Integer Linear Programming (BILP) with heuristics-based constraints. The heuristics were developed to search for a (local) optimal indoor anchor network that met the positioning accuracy requirements with the minimum number of anchors. Using the algorithm, we performed simulation for the optimal 2D and 3D positioning anchor network in a given floor plan. The simulation also took into account excessive delay due to the wall penetrations of UWB signals. In the simulation, the 2D positioning anchor network required 5 to 12 anchors to achieve 4 cm to 10 cm positioning accuracy (1 σ). The 3D positioning anchor network required 7 to 11 anchors to achieve 15 cm to 30 cm positioning accuracy. The anchor network taking into account wall excessive delays reduces the averaged number of wall penetrations in the user locations, which would minimize possible position biases when the wall excessive delay is uncompensated. We also discussed the optimality of the resultant anchor network geometries and concluded that they should be close to the global optimum. We also emphasize that the algorithm introduced in this paper could also apply to the positioning network with other ranging sources.

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Appendix

The figures of the resultant optimal anchor networks are shown in the next page.

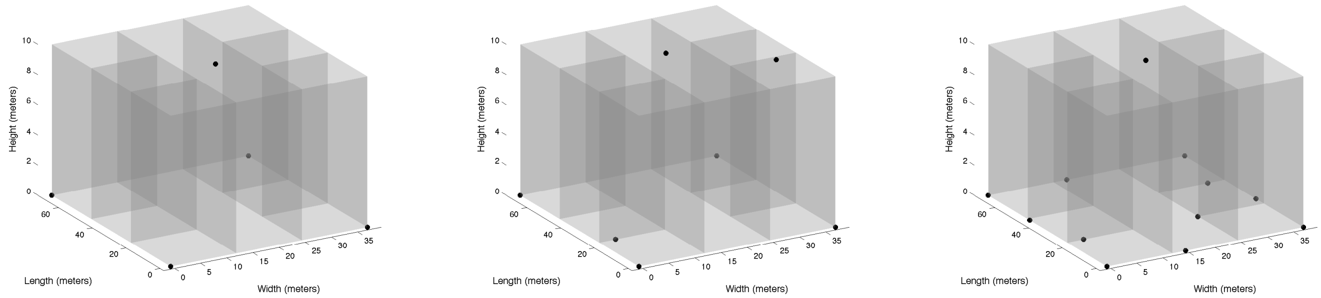

Fig. 6. 2D optimal positioning network without wall effects (Left: $\sigma_{\text{thr}} = 10$ cm, Middle: $\sigma_{\text{thr}} = 7$ cm, Right: $\sigma_{\text{thr}} = 4$ cm)

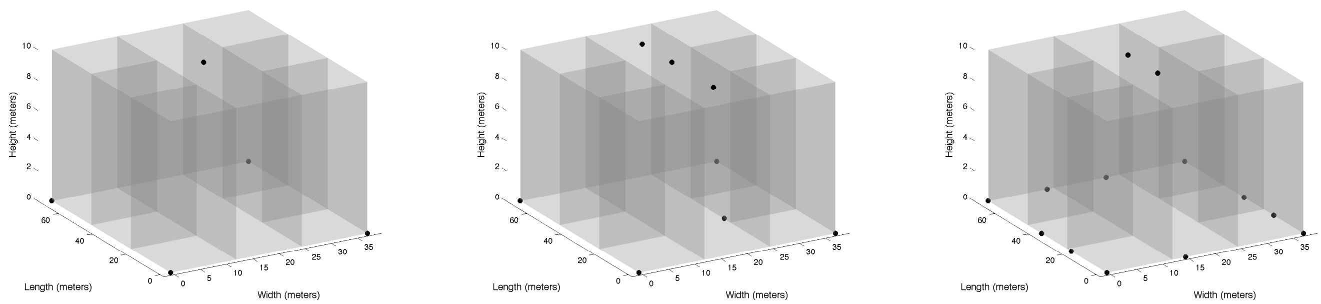

Fig. 7. 2D optimal positioning network with wall effects (Left: $\sigma_{\text{thr}} = 10$ cm, Middle: $\sigma_{\text{thr}} = 7$ cm, Right: $\sigma_{\text{thr}} = 4$ cm)

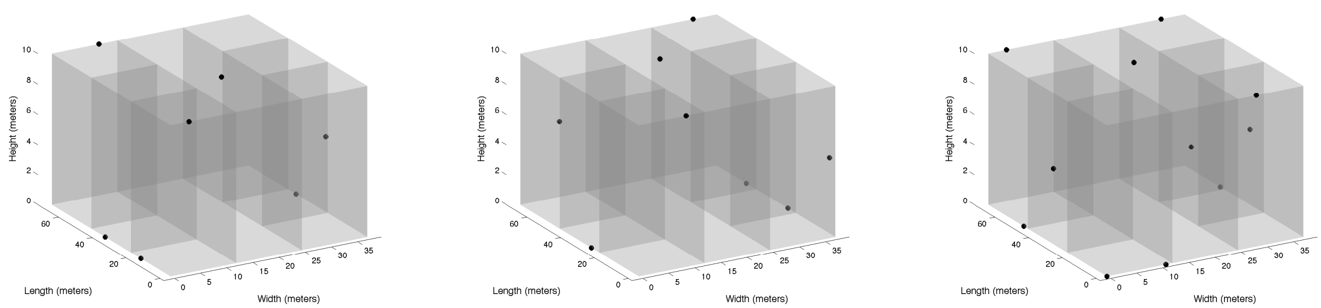

Fig. 8. 3D optimal positioning network without wall effects (Left: $\sigma_{\text{thr}} = 30$ cm, Middle: $\sigma_{\text{thr}} = 20$ cm, Right: $\sigma_{\text{thr}} = 15$ cm)

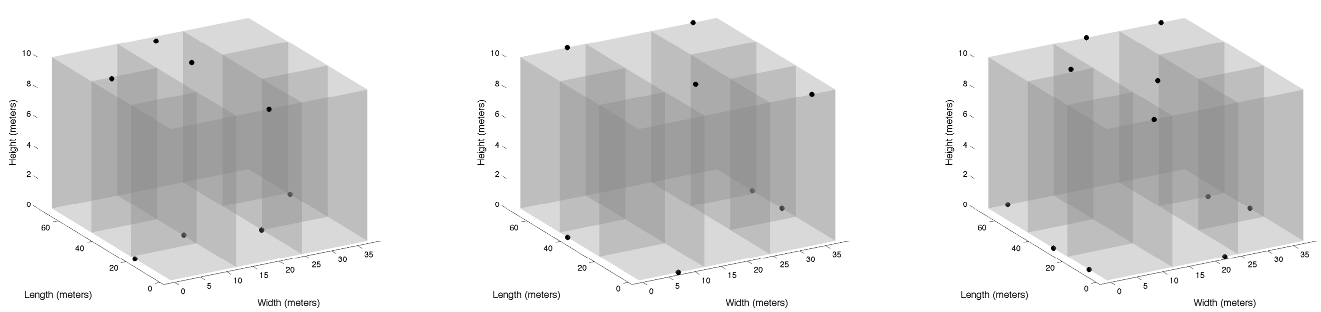

Fig. 9. 3D optimal positioning network with wall effects (Left: $\sigma_{\text{thr}} = 30$ cm, Middle: $\sigma_{\text{thr}} = 20$ cm, Right: $\sigma_{\text{thr}} = 15$ cm)